Scattering by a free electron (Thomson Scattering)

The total scattering cross section will be

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega$$

$$= \int r_0^2 \frac{1}{2}(1+\cos^2\phi)\, d\Omega$$

$$= \int_0^{\pi} r_0^2 \frac{1}{2}(1+\cos^2\phi)\, 2\pi \sin\phi\, d\phi$$

$$= r_0^2 \pi \int_0^{\pi} (\sin\phi + \cos^2\phi \sin\phi)\, d\phi$$

$$= r_0^2 \pi \int_0^{\pi} \left[-\cos\phi - \frac{\cos^3\phi}{3}\right]_0^{\pi} \quad \text{---------- (C)}$$

Result (C) will first of all derived by Thomson and so after his name it is called Thomson Scattering Cross section.

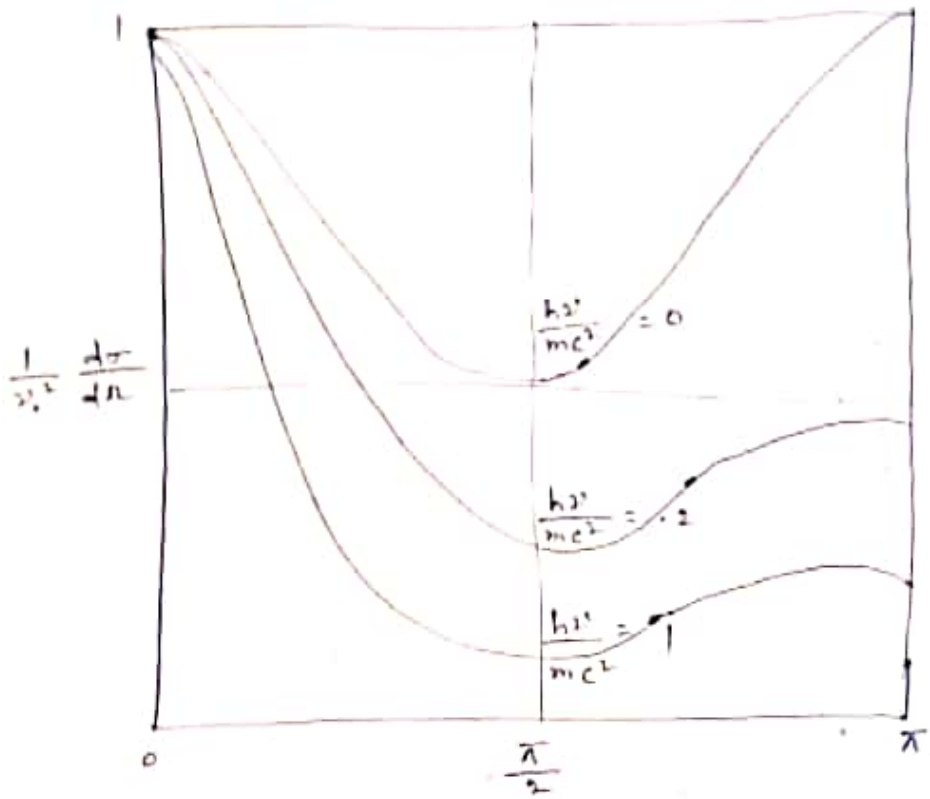
A quantum mechanical calculation carried by Klein and Nishina shows that deviations from Thomson result become significant for incident photon energy $h\nu$ which is comparable with or larger than the rest energies of the scattering electron mc^2. According to them

$$\sigma_{KN} = r_0^2 \left[\frac{8\pi}{3}\left(1 - \frac{2h\nu}{mc^2} + \cdots\cdots\right)\right]$$

and $$\sigma_{KN} = r_0^2 \left[\frac{\pi mc^2}{h\nu}\left[\log\left(\frac{2h\nu}{mc^2} + \frac{1}{2}\right)\right]\right]$$

These results are seen graphically

Scanned with CamScanner



From these curves it is clear that

(i) The scattering depends on the nature of incident radiations.

(ii) Quantum mechanical result approaches the classical one on the long wavelength side as the frequency $\nu = \frac{\omega}{2\pi}$ goes to zero.

(iii) The scattering is not symmetrical. In general the scattered radiation is more concentrated in the forward direction. ie $\phi = 0$.

Apart from these there is another feature of Thomson scattering which is modified by quantum considerations.

Scanned with CamScanner

Classically the scattered radiations has the same frequency as the incoming waves but quantum mechanical calculations shows that the frequency of the incoming waves but quantum mechanical calculation shows that the frequency of the scattered radiation is lesser than that of incoming waves depends on the angle of scattering. The relation between the wavelength of the scattered radiation at an angle ϕ and the incident radiation is

$$\lambda_s = \lambda_{s'} + \frac{h}{mc}(1 - \cos\phi)$$

Example — Calculate the value of and dimension of Thomson scattering cross section is given by.

Solution:- We know that Thomson Scattering cross section is given by

$$\sigma_T = r_0^2 \frac{8\pi}{3}$$

but as $r_0^2 = \dfrac{q^2}{4\pi\epsilon_0 mc^2}$

$$= \frac{(1.6\times10^{-19})^2}{4\pi\times9\times10^{-12}\times9.1\times10^{-31}\times(3\times10^8)^2} = 2.8\times10^{-15}\,m$$

So $\sigma_T = (2.8\times10^{-15})^2 \times \dfrac{8\pi}{3} = .665\times10^{-25}\,m$

Now as $\sigma = \dfrac{8\pi}{3} r_0^2 = \dfrac{8\pi}{3}\left[\dfrac{q^2}{4\pi\epsilon_0 mc^2}\right]^2$

and as $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r_0^2}$ i.e. $\left[\dfrac{q^2}{\epsilon_0}\right] = Fr^2 = ML^3T^{-2}$

$$\sigma = \left[\frac{ML^3T^{-2}}{ML^2T^{-2}}\right]^2 = L^2 = \text{Area}$$

Scanned with CamScanner

Scattering by a Bond electron (Rayleigh scattering)

Considering now the same charge as the previous section but elastically bound so that a restoring force $-m\omega_0^2 x$ is brought into play when it is displaced. We also asume that there is small amount of damping proportional to $\frac{dx}{dt}$ which may be produced in particle by collision or radiation. The equation of motion now becomes

$$m\frac{d^2x}{dt^2} = qE - m\gamma\frac{dx}{dt} - m\omega_0^2 x$$

where γ is the damping constant per unit mass.

Thus

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt}\omega_0^2 x = \frac{q}{m}E_0 e^{-i(\omega t - kx)} \quad \text{------} (1)$$

The solution of this differential equation consists two parts

(a) The complementry function: It is obtained by solving equation

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0 \quad \text{----------} (2)$$

Let the solution of equation (2) be

$$x = A^{\alpha t}$$

so that $\frac{dx}{dt} = A\alpha e^{\alpha t}$

Continue

(21-10-2020)

Scanned with CamScanner